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Top-Quark-Mass Enhancement in a Seesaw-Type Quark Mass Matrix *

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Abstract

We investigate the implications of a seesaw type mass matrix, i.e., $M_f \simeq m_L M_F^{-1} m_R$, for quarks and leptons f under the assumption that the matrices m_L and m_R are common to all flavors (up-/down- and quark-/lepton- sectors) and the matrices M_F characterizing the heavy fermion sectors have the form [(unit matrix) + b_f (a democratic matrix)] where b_f is a flavor parameter. We find that by adjusting the complex parameter b_f, the model can provide that $m_t \gg m_b$ while at the same time keeping $m_u \sim m_d$ without assuming any parameter with hierarchically different values between M_U and M_D. The model with three adjustable parameters under the "maximal" top quark mass enhancement can give reasonable values of five quark mass ratios and four KM matrix parameters.

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1. Introduction

One of the most mysterious facts in the quark mass spectrum is why top quark mass m_t is so much larger than the bottom quark mass m_b, while u quark mass m_u is of the order of d quark mass m_d. In the usual discussion of fermion masses, this drastic generation dependence of the mass splitting between members of each isomultiplet of quarks is attributed to an arbitrary hierarchy among the input parameters which is not completely satisfactory. It is therefore important to seek alternative ways to understand this feature. In this paper we argue that within the see-saw[1] type mass formula for quark masses discussed in the context of gauge models [2], a very simple explanation of this feature is obtained by imposing a specific universality ansatz for various flavor matrices. We then find that a slight generalization of this ansatz provides an extremely good fit to all the quark mass ratios and mixings.

Our starting point is the following specific see-saw type ansatz proposed by one of the authors [3] for quark and lepton mass matrices:

$$M_f = M_e^{1/2} O_f M_e^{1/2} \ , \tag{1.1}$$

where $M_e^{1/2} = \mathrm{diag}(\sqrt{m_e}, \sqrt{m_\mu}, \sqrt{m_\tau})$. Here, for the up-quark mass matrix M_u, the matrix O_f ($f = u$) is given by

$$O_f = \mathbf{1} + 3a_f X \ , \tag{1.2}$$

where $\mathbf{1}$ is a unit matrix and X is a democratic-type matrix [4]

$$X = \frac{1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \ , \tag{1.3}$$

which satisfies the relation $X^2 = X$. The up-quark mass matrix can then successfully give a quark mass ratio [3,5]

$$\frac{m_u}{m_c} \simeq \frac{3m_e}{4m_\mu} \ , \tag{1.4}$$

for a large value of the parameter a_u. The value of a_u is adjusted from the mass ratio m_c/m_t .

Stimulated by the phenomenological success of the mass matrix form (1.1) – (1.3), the authors [6] have applied the mass matrix form to down-quark mass matrix, by considering that the parameter a_d is complex. They have found that the value of a complex parameter a_d which fits the mass ratios m_d/m_s and m_s/m_b gives reasonable values of not only Kobayashi-Maskawa (KM) [7] matrix elements V_{ij} (i, j denote family indices) but also up-to-down-quark mass ratios m_u/m_d, m_c/m_s and m_t/m_b.

Suggested from the form (1.1), it may be expected that such phenomenological success will also be obtained in the context of a seesaw-type mass matrix

$$M_f \simeq m_L M_F^{-1} m_R \ , \tag{1.5}$$

with $m_L \propto m_R \propto M_e^{1/2}$ and $M_F \propto O_f^{-1}$. Here, the expression (1.5) is derived from the 6×6 mass matrix for fermions (f, F)

$$(\overline{f}_L \ \overline{F}_L) \left(\begin{array}{cc} 0 & m_L \\ m_R & M_F \end{array} \right) \left(\begin{array}{c} f_R \\ F_R \end{array} \right) + h.c. \ , \tag{1.6}$$

for the case of $O(M_F) \gg O(m_R), O(m_L)$, where$f = (f_1, f_2, f_3)$ are three family quarks and leptons, and $F = (F_1, F_2, F_3)$ are vector-like heavy fermions corresponding to f.

The re-interpretation of the model (1.1) based on the seesaw model (1.5) seems to be plausible because of the following reasons. The inverse of the matrix O_f with a simple form [(unit matrix)+(democratic-type matrix)] has also a simple form [(unit matrix)+(democratic-type matrix)], i.e.,

$$O_F \equiv O_f^{-1} = \mathbf{1} + 3b_f X \ , \tag{1.7}$$

where the complex coefficients a_f and b_f are related by

$$a_f = -b_f/(1+3b_f) \ . \tag{1.8}$$

In the mass matrix model (1.1), we need hierarchically different values [6] of the parameters a_f, i.e., $a_u = 28.65$ and $|a_d| = 0.4682$, in order to provide reasonable quark masses and KM mixings, while, as seen from (1.8), the values $|a_u| \gg 1$ and $a_d \simeq -1/2$ correspond to $b_u \simeq -1/3$ and $b_d \simeq -1$ in the inverse matrix (1.7), respectively. In the present paper, we are interested in such a model that M_u and

M_d are “almost” symmetric, i.e., they have almost the same structure and they take parameter values which are not so hierarchically different between M_u and M_d. The parameter ratio $|a_u/a_d| \simeq 60$ in the model (1.1) can be reduced to the ratio $|b_d/b_u| \simeq 3$ in (1.7).

However, when we consider a model (1.6) (not (1.5)) with $M_F \propto O_F$, one problem arises: Recently the CDF collaboration [8] has reported $m_t = 174 \pm 10^{+13}_{-12}$ GeV as top quark mass from $\overline{p}p$ collision data at $\sqrt{s} = 1.8$ TeV. On the other hand, the universal mass matrix m_L which breaks the $SU(2)_L$ gauge symmetry should be of the order $\Lambda_W = (\sqrt{2}G_F)^{-1/2}/\sqrt{2} = 174$ GeV$\sim m_t$, or less. Then, the approximate expression (1.4) for up-quarks is not valid any longer, because if (1.5) is valid, $O(m_L) \sim m_t$ means $M_U^{-1} m_R \sim O(1)$, so that it does not satisfy the condition $O(M_F) \gg O(m_R)$ for the validity of the seesaw expression (1.5). This is also understood from the fact that the limit $|a_u| \to \infty$ means the limit $b_u \to -1/3$ and the determinant of M_U becomes zero in the limit, so that the expansion of M_f in M_F^{-1} can not be a good approximation.

In this paper, we do not use the approximate relation (1.5), but calculate directly the 6×6 mass matrix (1.6). In Sect. 2, we will give the outline of our mass matrix model. In Sect. 3, we will give an expression of M_f which is valid in the limit of $b_f \to -1/3$, i.e., $\det M_F = 0$, instead of the well-known seesaw expression (1.5), and discuss the up-quark mass ratios which are expressed in terms of lepton mass ratios and our adjustable parameters (see the next section). In Sec. 4, we discuss the fermion mass spectra by numerically evaluating the 6×6 mass matrix. In Sect. 5, KM matrix parameters are discussed numerically. In the present model, under some basic assumptions (see Sects. 2 and 5), the parameter fitting for quark mass ratios and KM matrix parameters (5+4=9 observables) is done by three adjustable parameters k/K, b_d and β_d (see the next section for the definitions). We will find that the value of m_t takes the largest enhancement at $b_u = -1/3$, while the relations $m_u \sim m_d$ and (1.4) are kept. We can obtain reasonable values of quark mass ratios (not only m_u/m_c, m_c/m_t, m_d/m_s and m_s/m_b, but also m_u/m_d, m_c/m_s and m_t/m_b) and the KM matrix parameters, by taking $b_u = -1/3$ and $b_d \simeq -1$.

2. Outline of the model

In addition to the conventional quarks and leptons f_i, where f is the flavor index ($f = u, d, \nu$ and e denote up-quarks, down-quarks, neutrinos and charged leptons), and i is the family index ($i = 1, 2, 3$), We consider vector-like fermions F_i

correspondingly to f_i. These fermions belong to $f_L = (2,1)$, $f_R = (1,2)$, $F_L = (1,1)$ and $F_R = (1,1)$ of $\mathrm{SU(2)}_L\times\mathrm{SU(2)}_R$. A "would-be" seesaw mass matrix for the fermions (f, F) is given by (1.6). Gauge models which realize the mass matrix form (1.6) have been proposed by many authors [2]. Although the interest of most authors is how to embed the model (1.6) into a unification model in the framework of gauge theory, our interest is how to give realistic quark mass spectra and family mixing from the phenomenological point of view.

Suggested by the phenomenological success of the model (1.1), we assume the following mass matrix [9]

$$M = \begin{pmatrix} 0 & m_L \\ m_R & M_F \end{pmatrix} = m_0 \begin{pmatrix} 0 & Z \\ kZ & KO_F \end{pmatrix} , \tag{2.1}$$

where the matrices m_L and m_R (i.e., m_0, h and the matrix Z) are common to all of $f = u, d, \nu, e$, and only M_F depends on flavors f through the complex parameter b_f. Hereafter, we denote the complex parameter b_f in (1.7) as $b_f e^{i\beta}$ (b_f is real and $|\beta_f| \leq \pi/2$) in (2.2) below. The vector-like fermions F acquire large masses M_F at an energy scale $\mu = m_0 K$. We consider that the energy scale $m_0 K$ is not as large as the ground unification scale, but an intermediate energy scale. At the present stage, the origin of the democratic form

$$O_F = \mathbf{1} + 3b_f e^{i\beta_f} X = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + b_f e^{i\beta_f} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} , \tag{2.2}$$

is an open question. We may attribute the origin of the democratic term X to a permutation symmetry S_3 [10], a BCS-like mechanism [11], a composite model based on the analogy of hadronic π^0-η-η' mixing [12], and so on. In the present phenomenological analysis, we do not discuss its origin moreover.

The present model is left-right symmetric except for $k \neq 1$. At an energy scale $\mu = m_0 k$ ($\mu = m_0$) at which $\mathrm{SU(2)}_R$ ($\mathrm{SU(2)}_L$) is broken, the mass term $\overline{F}_L m_R f_R$ ($\overline{f}_L m_L F_R$) appears, so that we consider $k \sim m(W_R)/m(W_L)$. The relation $m_L = m_R/k = m_0 Z$ is merely a phenomenological working hypothesis. The matrix Z takes a diagonal form

$$Z = \mathrm{diag}(z_1, z_2, z_3) , \tag{2.3}$$

with the normalization condition $z_1^2 + z_2^2 + z_3^2 = 1$. (In other words, in the family basis in which Z is diagonal, we have assumed that the matrix O_F is given by (2.2)). For the charged leptons, since $m_\tau \ll m_0 \sim m_W$, it is clear that the seesaw expression $M_e = m_0(k/K)ZO_F^{-1}Z$ is well satisfied, so that we can fix the parameter z_i as

$$\frac{z_1}{\sqrt{m_e}} = \frac{z_2}{\sqrt{m_\mu}} = \frac{z_3}{\sqrt{m_\tau}} = \frac{1}{\sqrt{m_e + m_\mu + m_\tau}} \ . \tag{2.4}$$

Here, we have assumed $b_e = 0$ according to the phenomenological success [3] of the model (1.1). In the present paper, we do not discuss why z_i are given by the relation (2.4), because the purpose of the present paper is to study quark mass ratios and KM matrix parameters phenomenologically, so that charged lepton masses are regarded as inputs in the numerical estimates. Since the evolution effects of fermion mass ratios (not the absolute values) from $\mu = m_0 K$ to $\mu = m_0$ are, at most, several percent, for simplicity, we use the values of z_i which are fixed by using the formula (2.4) with the observed charged lepton masses [13].

For the case of $K \gg k \gg 1$, the quark mass ratios and the KM matrix parameters (nine observables) are described by five real parameters k/K (not k and K separately), b_u, β_u, b_d and β_d. As we will discuss in Sections 3 and 4, the maximal top-quark-mass enhancement occurs at $b_u = -1/3$ and $\beta_u = 0$. We will put an ansatz of "maximal top-quark-mass enhancement", so that we will fix the parameters b_u and β_u to $b_u = -1/3$ and $\beta_u = 0$. The numerical fitting for the nine observables is then tried by adjusting only three parameters k/K, b_d and β_d. However, as will be discussed in Sect. 5, a straightforward application of the mass-matrix model (2.1) cannot lead to reasonable predictions of the KM matrix parameters. We will therefore introduce a sign factor by replacing $m_L = m_0 Z$ in (2.1) by $m_L^f = m_0 P_f Z$, where $P_u = \mathrm{diag}(1, 1, 1)$, while $P_d = \mathrm{diag}(1, 1, -1)$. The adjustable parameters are still three, i.e., k/K, b_d and β_d. The phase matrices P_f do not affect the discussion of the mass spectrum. For a time being in Sects. 3 and 4, we will neglect the phase matrices P_f.

3. Expression of M_f in the case of $b_f \simeq -1/3$

One of the purposes in the present paper is to obtain a reliable expression of M_f in the case of $b_f \simeq -1/3$, because the case leads to $\det M_F \simeq 0$, so that the seesaw expression (1.5) which is obtained by expanding it in M_F^{-1} is not valid any

longer.

As shown in Appendix, in general, the transformation of the 6×6 mass matrix M into

$$U_L M U_R^\dagger \equiv U_L \left(\begin{array}{cc} M_{11} & M_{12} \\ M_{21} & M_{22} \end{array} \right) U_R^\dagger = M' \equiv \left(\begin{array}{cc} M'_{11} & 0 \\ 0 & M'_{22} \end{array} \right) \ , \tag{3.1}$$

is done by the following two 6×6 unitary matrices,

$$U_L = \left(\begin{array}{cc} (1+\rho_L\rho_L^\dagger)^{-1/2} & (1+\rho_L\rho_L^\dagger)^{-1/2}\rho_L \\ -(1+\rho_L^\dagger\rho_L)^{-1/2}\rho_L^\dagger & (1+\rho_L^\dagger\rho_L)^{-1/2} \end{array} \right) \tag{3.2}$$

and U_R with $L \leftrightarrow R$ in (3.2). The so-called seesaw expression $M'_{11} \equiv M_f \simeq m_L M_F^{-1} m_R$ is obtained by expanding M'_{11} in M_F^{-1}. Since our mass matrix (2.1) is not Hermitian, for evaluating the KM matrix (family mixing of left-handed fermions), it is useful to define the 3×3 Hermitian matrix H_f:

$$H_f \equiv M'_{11} M'^\dagger_{11} = (1+\rho_L\rho_L^\dagger)^{-1/2} \widetilde{H}_f (1+\rho_L\rho_L^\dagger)^{+1/2} \ . \tag{3.3}$$

As seen in (A.22), (A.24) and (A.27), the matrix $\widetilde{H}_f$ is given by

$$\widetilde{H}_f \equiv \rho_L m_R \rho_R m_L^\dagger = (m_L + \rho_L M_F) m_L^\dagger \ , \tag{3.4}$$

and it satisfies the following equation:

$$\widetilde{H}_f^2 m_L^{\dagger -1} - \widetilde{H}_f m_L^{\dagger -1} \left(M_F^\dagger M_F + m_L^\dagger m_L + M_F^{-1} m_R m_R^\dagger M_F \right) + m_L M_F^{-1} m_R m_R^\dagger M_F = 0 \ . \tag{3.5}$$

Our interest is in the expression of $\widetilde{H}_f$ in the case of $\det M_F \simeq 0$. However, since it is hard to obtain the general formulation in such the case, we confine ourselves to investigating the special form (2.1) with (2.2).

For the investigation of the case of $b_u \simeq -1/3$, it is convenient to define the parameter

$$3\varepsilon \equiv \Delta b = b + \frac{1}{3} \ . \tag{3.6}$$

Then, the matrix O_F is represented by

$$O_F = Y + \varepsilon X \ , \tag{3.7}$$

where

$$Y = \mathbf{1} - X \ , \tag{3.8}$$

and the matrices X and Y satisfy the relations $X^2 = X$, $Y^2 = Y$, and $XY = YX = 0$ from the definitions (1.3) and (3.8), so that the inverse of O_F, (3.7), is given by

$$O_F^{-1} = Y + X/\varepsilon \ . \tag{3.9}$$

For the case of $(k/K)^2 \ll \varepsilon^2 \ll 1$, from the equation (3.5), we obtain

$$\widetilde{H}_f \simeq m_0^2 \left(\frac{k}{K}\right)^2 Z \left(Y + \frac{1}{\varepsilon}X\right) Z^2 \left(Y + \frac{1}{\varepsilon}X\right) Z \ , \tag{3.10}$$

which corresponds to the well-known seesaw expression $M_f \simeq m_0(k/K)ZO_F^{-1}Z$.

For a general case, we assume an approximate form

$$\widetilde{H}_u \simeq m_0^2 Z \left(\frac{k}{K}Y + xX\right) Z^2 \left(\frac{k}{K}Y + xX\right) Z \ , \tag{3.11}$$

from an analogy to the form (3.10). By substituting (3.11) into (3.5), we find

$$x \simeq \left[\frac{\varepsilon}{2k/K} + \sqrt{\frac{1}{3} + \left(\frac{\varepsilon}{2k/K}\right)^2}\ \right]^{-1} \ . \tag{3.12}$$

For $\varepsilon^2 \gg (k/K)^2$, (3.12) reproduces (3.10). For $\varepsilon^2 \ll (k/K)^2$, we obtain

$$\widetilde{H}_u \simeq 3m_0^2 Z \left(X + \frac{1}{\sqrt{3}}\frac{k}{K}Y\right) Z^2 \left(X + \frac{1}{\sqrt{3}}\frac{k}{K}Y\right) Z \ . \tag{3.13}$$

This expression (3.13) is the expression which should be used in the case of $\det M_F \simeq 0$ as a substitute for the well-known seesaw expression (3.10).

The mass eigenvalues are calculated from $\mathrm{Tr}H_u = \mathrm{Tr}\widetilde{H}_u$, $((\mathrm{Tr}H_u)^2 - \mathrm{Tr}H_u^2)/2 = ((\mathrm{Tr}\widetilde{H}_u)^2 - \mathrm{Tr}\widetilde{H}_u^2)/2$ and $\det H_u = \det\widetilde{H}_u$. We obtain up-quark masses

$$m_u \simeq \frac{3}{2} z_1^2 \frac{k}{K} m_0 \ , \quad m_c \simeq 2 z_2^2 z_3^2 \frac{k}{K} m_0 \ , \quad m_t \simeq \frac{1}{\sqrt{3}} \frac{1}{\sqrt{1 + 27(\Delta b)^2 (K/k)^2}} m_0 \ , \tag{3.14}$$

from (3.11) and (3.12), where $\varepsilon = \Delta b/3$ (3.6). We find that the relation (1.4) is also valid in the case $(\Delta b)^2 \ll (k/K)^2 \ll 1$, even in the limit of $b_u = -1/3$.

4. Numerical study of quark mass ratios

Numerical evaluation of the eigenvalues of the 6×6 mass matrix (2.1) can easily be done with the help of a computer. Numerical study is helpful for checking analytical calculations based on the formalism of the previous section. In Fig. 1, in order to give an overview of the mass spectrum in our mass matrix model, we illustrate the light fermion mass spectrum m_i^f $(i = 1, 2, 3)$ versus the parameter $b_f e^{i\beta_f}$. Here, we have taken $k = 10$ and $K/k = 50$ as a trial. (The choices of k and K/k are discussed later.) In order to fix the values of the parameters z_i at $b_e = 0$, we have used the observed charged leptons masses [13] as inputs.

The spectrum for the case of $\beta_f = 0$ (solid lines) shows the following characteristics:

(1) The third fermion mass is sharply enhanced at $b_f = -1/3$.

(2) Level crossing (mass degeneration) occurs at $b_f = -1/2$ and $b_f = -1$.

These characteristics become mild when β_f takes a sizable value (dashed lines).

For comparison, we list the observed running quark mass values (in unit of GeV) [14] at $\mu = \Lambda_W \equiv (\sqrt{2} G_F)^{-1/2} = 174$ GeV:

$$\begin{array}{lll} m_u = 0.00230 \pm 0.00045 \ , & m_c = 0.612^{+0.010}_{-0.023} \ , & m_t = 166^{+21}_{-26} \ , \\ m_d = 0.00406 \pm 0.00045 \ , & m_s = 0.082 \pm 0.014 \ , & m_b = 2.874^{+0.012}_{-0.023} \ . \end{array} \tag{4.1}$$

In the previous section, we have showed that the up-quark mass ratio m_u/m_c is given by (1.4) in the limit $\varepsilon \ll (k/K)^2 \ll 1$, see (3.14). The relation can be checked by a numerical study. We find that the ratio m_c/m_u at a fixed K/k is insensitive to the choice of k, for $k \geq 10$. Also, the ratio is insensitive to the parameters K/k and Δb_u for large K/k; for example, $m_c/m_u = 260.8, 260.8, 259.2$, and 259.2, for $(K, k, \Delta b_u) = (10^3, 10, 0), (10^5, 10, 0)$, $(10^3, 10, 0.003)$ and $(10^5, 10, 0.003)$, respectively, while $(m_c/m_u)_{exp} = 266^{+70}_{-49}$. Thus, we conclude that the relation (1.4) is valid almost independently of the values of k and K/k for the case of $K \gg k \gg 1$.

Next, we study the up-quark mass ratio m_t/m_c. We find that the ratio is also insensitive to the value of k for $k \geq 10$. Therefore, we illustrate the behavior of m_t/m_c versus K/k for the case of $k = 10$ in Fig. 2. It is noticeable that, for $\Delta b_u \simeq +0.00388$ and $\Delta b_u \simeq -0.00362$, the ratio m_t/m_c comes near the experimental value

$(m_t/m_c)_{exp} \simeq 271$ as $K/k \to \infty$. For the case $|\Delta b_u| \geq 0.005$, we cannot fit the ratio m_c/m_t suitably, so that the case is ruled out. For $\Delta b_u = 0$, the ratio m_t/m_c increases linearly in K/k. In order to fit the prediction to the experimental value of $(m_t/m_c)_{exp} = 271 \pm 46$, we need $K/k = 50 \pm 8$ for the case $\Delta b_u = 0$ whereas we find $K/k = (2.0^{+\infty}_{-1.3}) \times 10^2$ for the cases $\Delta b_u = +0.00388$ and $\Delta b_u = -0.00362$.

Although a scenario with $\Delta b_u \simeq \pm 0.004$ and $K/k > 2 \times 10^2$ seems to be attractive because the ratio m_t/m_c can be fitted insensitive to K/k, we do not adopt this scenario because of the following consideration of the absolute value of m_t. In Fig. 3, we show the behavior of m_t/m_0 versus K/k. Since the ratio is again insensitive to the value of k for $k \geq 10$, we illustrate the case of $k = 10$. In the limit of $b_u = -1/3$, the value m_t/m_0 is almost constant, i.e., $m_t/m_0 \simeq 1/\sqrt{3}$ (a) as we have shown in (3.14). On the other hand, as seen in Fig. 3, the case (c) $\Delta b_u \simeq \pm 0.004$ gives $m_t/m_0 < 0.161$ for $K/k > 2 \times 10^2$. If we consider that the mass matrix m_L originates from the couplings to an $\mathrm{SU}(2)_L$ doublet Higgs boson ϕ_L with the vacuum expectation value (VEV) $\langle \phi_L^0 \rangle_0 = v_0 = \Lambda_W = 174$ GeV, the Yukawa coupling constants y_{Li} with fermions $\overline{f}_{Li} F_{Ri}$ are given by $y_{Li} = z_i m_0 / v_0$. Therefore, a small value of m_t/m_0 means a large value of $y_{L3} = z_3 (m_0/m_t)(m_t/v_0)$. The value $m_t/m_0 = 0.161$ corresponds to $y_{L3} = 6.03(m_t/v_0)$. Such a large value may be unfavorable from the point of view of the perturbative electroweak theory. Hereafter, we adopt the ansatz of the "maximal top-quark-mass enhancement", i.e., $b_u = -1/3$ (solid lines in Figs. 2 and 3), and we fix the parameter K/k to $K/k = 50$ from the observed ratio of m_t/m_c.

On the other hand, the down-quark masses are given by adjusting two parameters b_d and β_d. As seen in Fig. 1, the case of $b_d \simeq -1$ is favorable because it can give reasonable predictions not only for m_b/m_s and m_s/m_d, but also for m_d/m_u. The ratios m_s/m_d and m_b/m_s versus b_d and β_d are illustrated in Fig. 4 for the case of $K/k = 50$ and $k = 10$. As far as we see in Fig. 4, the cases $b_d = -1.1 \sim -1$ with $\beta_d = -20^\circ \sim -16^\circ$ are favorable. Considering the present experimental uncertainty of quark mass values, hereafter, we simply adopt the integral solution $b_d = -1$ for further numerical estimates.

For the case of $b_d \simeq -1$ and $1 \gg \beta_d^2 \neq 0$, down-quark masses are given by

$$m_d \simeq z_1^2 \frac{1}{\beta_d} \frac{k}{K} m_0 \ , \quad m_s \simeq z_2^2 z_3^2 \beta_d \frac{k}{K} m_0 \ , \quad m_b \simeq \frac{1}{2} \frac{k}{K} m_0 \ . \tag{4.2}$$

In the present model, the up-to-down quark mass ratio m_u/m_d is given by

$$\frac{m_u}{m_d} \simeq 3\frac{m_s}{m_c} \simeq \frac{3}{2}\beta_d \ , \tag{4.3}$$

so that the ratios m_u/m_d and m_s/m_c can be fitted independently of m_t/m_c (i.e., K/k) by adjusting the parameter β_d.

When we take $b_d = -1.0$ and $\beta_d = -18^\circ$ (and $k = 10$ and $K/k = 50$), we can obtain reasonable quark mass values:

$$\begin{array}{lll} m_u(\Lambda_W) = 0.00234 \text{ GeV} \ , & m_c(\Lambda_W) = 0.610 \text{ GeV} \ , & m_t(\Lambda_W) = 166 \text{ GeV} \ , \\ m_d(\Lambda_W) = 0.00475 \text{ GeV} \ , & m_s(\Lambda_W) = 0.0923 \text{ GeV} \ , & m_b(\Lambda_W) = 3.01 \text{ GeV} \ , \end{array} \tag{4.4}$$

where we have taken $m_0(\Lambda_W) = 288$ GeV to have $m_t(\Lambda_W) = 166$ GeV.

So far, except for (4.4), we have discussed only quark mass ratios and not the absolute values, because the ratios are comparatively insensitive to the evolution from $\mu = m_0 K$ to $\mu = m_0$. The common value $m_0(\Lambda_W) = 288$ GeV does not give the absolute magnitudes of the charged lepton masses, $(k/K)m_0 = m_\tau + m_\mu + m_e$. We find

$$\left. \frac{(m_0 k/K)_q}{(m_0 k/K)_\ell} \right|_{\mu=\Lambda_W} = 3.05 \ , \tag{4.5}$$

where $(m_0 k/K)_{q(\ell)}$ denotes the value of $m_0 k/K$ in the quark (lepton) sector. It is not likely that the factor 3.1 comes only from the evolution from $\mu = m_0 K$ to the present scale $\mu = \Lambda_W$. Since we consider the case where the parameters m_0 and k (i.e., m_L and m_R) are universal for all flavors $f = u, d, \nu, e$, the discrepancy (4.5) should come from the difference in K between the quark- and lepton-sectors, i.e., $K_q \neq K_\ell$. Although it is possible that the coupling constants of the colored heavy fermions with Higgs bosons which generate the democratic-type matrix (2.2) are smaller than that of the colorless heavy fermions by a factor 1/3, i.e., $K_\ell/K_q = 3$, we do not discuss the origin of $K_\ell/K_q = 3$ in the present paper. In the present model, we practically consider that m_L and m_R are universal for quarks and leptons, while M_F are not so, and $K_u = K_d \equiv K_q \neq K_\nu = K_e \equiv K_\ell$. Hereafter, we denote K_q simply as K.

Similarly, with the same parameter values as in (4.4), the heavy quark masses are given as follows:

$$\begin{array}{lll} m_4^u(\Lambda_W) = 1.66 \text{ TeV} \ , & m_5^u(\Lambda_W) = 144 \text{ TeV} \ , & m_6^u(\Lambda_W) = 144 \text{ TeV} \ , \\ m_4^d(\Lambda_W) = 144 \text{ TeV} \ , & m_5^d(\Lambda_W) = 144 \text{ TeV} \ , & m_6^d(\Lambda_W) = 298 \text{ TeV} \ . \end{array} \tag{4.6}$$

These numerical results are also obtained from the approximate relations for $b_u = -1/3$ and $b_d = -1$:

$$m_4^u \simeq (k/\sqrt{3}) m_0 \ , \quad m_5^u \simeq m_6^u \simeq K m_0 \ , \tag{4.7}$$

$$m_4^d \simeq m_5^d \simeq K m_0 \ , \quad m_6^d \simeq 2\sqrt{1 + 3\beta_d^2/4} K m_0 \ . \tag{4.8}$$

Note that the fourth up-quark u_4 becomes considerably lighter than the other heavy quarks, at the cost of the enhancingthe top-quark mass. The absolute magnitudes the heavy quark masses in (4.6) should not be taken solidly, because they depend on both k and K. We have chosen $K/k = 50$ in order to fit m_t/m_c, but the choice $k = 10$ was only a trial choice, because the predictions for light fermions (quarks and leptons) are insensitive to the value of k. Only constraint on the value k comes from the relation $k \sim m(W_R^\pm)/m(W_L^\pm)$. The present lower bound of the right-handed weak boson mass $m(W_R)$ is given in Ref. [15], so that we cannot choose too small value of k. Since m_4^u is of the order of $k m_0$, as seen in (4.7), we can expect to observe the fourth up-quark at the energy scale where the right-handed weak bosons W_R are observed.

5. KM matrix parameters

In the present model, the parameter fitting for five quark-mass ratios and four KM matrix parameters is done by five parameters, k/K (not k and K), b_u, β_u, b_d and β_d. When we adopt the ansatz of "maximal top-quark-mass enhancement", we have fixed the parameters b_u and β_u to $b_u = -1/3$ and $\beta_u = 0$, and the remaining adjustable parameters are k/K, b_d and β_d. We have pointed out that the relation between up-quark mass ratio m_u/m_c and m_e/m_μ, (1.4), is satisfied independently of these parameters for the case $b_u \simeq -1/3$. The parameter K/k was fixed to $K/k = 50$ from the observed up-quark mass ratio m_t/m_c, see Fig. 2. In the previous section, we have shown that the remaining two parameter b_d and β_d can be fitted to three observed quark mass ratios m_d/m_s, m_s/m_b and m_u/m_d reasonably (see Fig. 4). Then, our final task in the present phenomenological study is to check

whether these parameter values can also give reasonable predictions for the four KM matrix parameters.

The KM matrix V is given by $V = U_u U_d^\dagger$, where U_q $(q = u, d)$ are the unitary matrices to diagonalize the light fermion mass matrices $M_f M_f^\dagger$, where $M_f \equiv M'_{11}$ $(f = u, d)$ defined by (3.2). Unfortunately, our parameter values $K/k \simeq 50$, $b_d \simeq -1$ and $\beta_d \simeq -18^\circ$ give rise to the KM matrix parameters far away from the observed values [13]. Therefore, we must slightly modify our model.

So far, we have assumed that the matrices m_L and m_R are universal for up- and down-sectors. However, in the present section, let us distinguish the matrix m_L in the up-quark sector, $m_L^u = m_0 Z_u$, from that in down-quark sector, $m_L^d = m_0 Z_d$. We assume that Z_u and Z_d are given by $Z_q = P_q Z$ $(q = u, d)$, where Z is given by (2.3) and (2.4), and P_q are phase matrices. (It is not essential whether we also assume a similar modification on m_R or not, because the KM matrix is related only to the family mixing among the left-handed fields.) Such a modification does not change our predictions on the fermion masses in Sects. 3 and 4, while the KM matrix V is changed into the following expression:

$$V = U_u P U_d^\dagger \ , \tag{5.1}$$

where U_q $(q = u, d)$ are unitary matrices to diagonalize the unchanged matrices $M_f M_f^\dagger$ (i.e., in the case of $P_u = P_d = \mathbf{1}$), and $P = P_u P_d^\dagger$. In general, the phase matrix P can have two independent phase parameters such as $P = \mathrm{diag}(1, e^{i\delta_2}, e^{i\delta_3})$. However, since we do not want more adjustable parameters, we examine a simpler ansatz that the phase matrix P is real, i.e., $\delta_i = 0$ or π. Thus, we keep three adjustable parameters, k/K, b_d and β_d, at the cost of putting the additional ansatz on P.

As a result, we find that only for the case

$$P = \mathrm{diag}(1, 1, -1) \ , \tag{5.2}$$

we can obtain reasonable values of $|V_{us}|$, $|V_{cb}|$ and $|V_{ub}|$. We show $|V_{us}|$, $|V_{cb}|$ and $|V_{ub}|$ versus β_d in Fig. 5. The same parameter values as in (4.4), $K/k = 50$, $b_d = -1$ and $\beta_d = -18^\circ$, give reasonable predictions

$$|V_{us}| = 0.220 \ , \quad |V_{cb}| = 0.0598 \ , \quad |V_{ub}| = 0.00330, \quad |V_{td}| = 0.0155,$$

$$J = -3.18 \times 10^{-5} \ , \tag{5.3}$$

where J is the rephasing invariant [16] $J = \mathrm{Im}(V_{cb}V_{us}V_{cs}^*V_{ub}^*)$. Although the origin of the phase inversion $P = \mathrm{diag}(1, 1, -1)$ is not clear and the predicted value of V_{cb} is somewhat large, it is a noticeable feature of the present model that the parameters which were fixed by the observed quark-mass ratios can roughly give reasonable predictions for all the KM matrix parameters.

6. Conclusions

In conclusion, we have demonstrated that the seesaw-type mass matrix (2.1) with M_F given by (2.2) can give top-quark-mass enhancement without assuming any parameters with hierarchically different values between M_U and M_D, i.e., with $b_u \simeq -1/3$ and $b_d \simeq -1$. The enhancement $m_t/m_b \gg 1$ comes from the fact that the democratic part X in the inverse matrix M_F^{-1} in (1.2), is enhanced as to $b_f \to -1/3$ because $|a_f| \to \infty$ in the limit as seen in (1.8). On the other hand, the result $m_u \sim m_d$ comes from the feature that the democratic-type mass matrix can give rise to a large mass only to the third family, i.e., the effect of $|a_u| \to \infty$ contributes mainly to m_t.

In the present model, the parameter fitting for the five quark mass ratios and the four KM matrix parameters has been done by five parameters k/K (not k and K separately), b_u, β_u, b_d and β_d. (The parameters z_i were fixed by charged lepton masses.) When we adopt the ansatz of "maximal top-quark-mass enhancement", the parameters b_u and β_u are fixed to $b_u = -1/3$ and $\beta_u = 0$, and the remaining adjustable parameters are k/K, b_d and β_d. The parameter K/k is then fixed by the observed up-quark-mass ratio m_t/m_c to be $K/k = 50$. The remaining two parameters b_d and β_d are then free parameters by which four quark mass ratios m_u/m_c, m_d/m_s, m_s/m_b and m_u/m_d, and four KM parameters are fitted. As shown in Sects. 4 and 5, by choosing $b_d \simeq -1$ and $\beta_d \simeq -18^\circ$, we have obtained reasonable fitting for the quark-mass ratios, and also for the KM matrix parameters with the ansatz (5.2).

A few remarks are in order.

In the present model, flavor changing neutral currents (FCNC) can, in principle, appear. However, the FCNC due to the $\mathrm{SU}(2)_L$ ($\mathrm{SU}(2)_R$) doublet Higgs boson exchange through f-F mixing are highly suppressed by a GIM-like mechanism [17]. The FCNC due to the Z-boson exchange through f-F mixing are also suppressed because the effective coupling constants are order of $1/K$ (we can find that those are of the order of 10^{-8} in the case of $k = 10$), so that the FCNC rare decay modes

are suppressed by 10^{-16}.

The CP violating phases come only from the heavy fermion mass matrix M_F, i.e., from the parameter β_f. In the up-quark sector, the parameter β_u must be $\beta_u = 0$, because the top quark mass enhancement becomes mild when $\beta_u \neq 0$. On the other hand, if $\beta_d = 0$, we cannot fit down-quark mass ratios m_d/m_s and m_s/m_b for any values of k/K and b_d. We must choose a sizable value of β_d. Thus, in our model, the CP violating phase in quarks comes only from the down-quark sector M_D.

In the present paper, we have discussed a seesaw mass matrix model with the form of $M_F = m_0 K O_F$ given by (2.2). As far as the phenomenological predictions are concerned, we can choose other family-basis, for example, a rather simple form of O_F

$$O_F = \mathbf{1} + 3 b_f e^{i\beta_f} \mathrm{diag}(0,0,1) \ , \tag{6.1}$$

instead of the democratic form (2.2). However, in order to obtain reasonable predictions of quark mass ratios and KM matrix parameters, the matrix Z cannot be a diagonal form such as in (2.3), and it must be given by

$$Z = \frac{1}{6} \left(\begin{array}{ccc} 3(z_2+z_1) & -\sqrt{3}(z_2-z_1) & -\sqrt{6}(z_2-z_1) \\ -\sqrt{3}(z_2-z_1) & 4z_3+z_2+z_1 & -\sqrt{2}(2z_3-z_2-z_1) \\ -\sqrt{6}(z_2-z_1) & -\sqrt{2}(2z_3-z_2-z_1) & 2(z_3+z_2+z_1) \end{array} \right) \ , \tag{6.2}$$

where z_i are given by (2.4). Which family basis is reasonable is not essential as far as we discuss only the fermion masses and KM mixing parameters, but it will become important for model-building.

We believe that our phenomenological mass-matrix model is worth serious attention, not only because it has fewer adjustable parameters than conventional models do, but also because it gives $m_t \gg m_b$ and $m_u \sim m_d$ simultaneously despite its "almost" up-down symmetric mass matrices (i.e., b_u/b_d is not so large as m_t/m_b).

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Appendix: Diagonalization of $2n \times 2n$ matrix

The transformation of $2n \times 2n$ matrix

$$M = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \tag{A.1}$$

into

$$M' \equiv \begin{pmatrix} M'_{11} & 0 \\ 0 & M'_{22} \end{pmatrix} \tag{A.2}$$

is done by two $2n \times 2n$ unitary matrices,

$$U_L = \begin{pmatrix} (1+\rho_L\rho_L^\dagger)^{-1/2} & (1+\rho_L\rho_L^\dagger)^{-1/2}\rho_L \\ -(1+\rho_L^\dagger\rho_L)^{-1/2}\rho_L^\dagger & (1+\rho_L^\dagger\rho_L)^{-1/2} \end{pmatrix} \tag{A.3}$$

and U_R with $L \leftrightarrow R$ in (A.3) as

$$M' = U_L M U_R^\dagger \ , \tag{A.4}$$

where M_{ij}, M'_{ij}, ρ_L, ρ_R are $n \times n$ matrices.

The conditions $M'_{12} = 0$ and $M'_{21} = 0$ lead to the relations

$$M_{12} - M_{11}\rho_R + \rho_L M_{22} - \rho_L M_{21}\rho_R = 0 \ , \tag{A.5}$$

and

$$M_{21} + M_{22}\rho_R^\dagger - \rho_L^\dagger M_{11} - \rho_L^\dagger M_{12}\rho_R^\dagger = 0 \ , \tag{A.6}$$

respectively, which lead to

$$\rho_R = (M_{11} + \rho_L M_{21})^{-1}(M_{12} + \rho_L M_{22}) \ , \tag{A.7}$$

$$\rho_R = (M_{21}^\dagger - M_{11}^\dagger \rho_L)(M_{12}^\dagger \rho_L - M_{22}^\dagger)^{-1} \ . \tag{A.8}$$

By eliminating ρ_R from (A.7) and (A.8), we obtain

$$(M_{12} + \rho_L M_{22})(M_{12}^\dagger \rho_L - M_{22}^\dagger) = (M_{11} + \rho_L M_{21})(M_{21}^\dagger - M_{11}^\dagger \rho_L) \ , \tag{A.9}$$

or

$$\begin{aligned} &M_{11}M_{21}^\dagger + M_{12}M_{22}^\dagger - (M_{11}M_{11}^\dagger + M_{12}M_{12}^\dagger)\rho_L \\ &+\rho_L(M_{21}M_{21}^\dagger + M_{22}M_{22}^\dagger) - \rho_L(M_{21}M_{11}^\dagger + M_{22}M_{12}^\dagger)\rho_L = 0 \ , \end{aligned} \tag{A.10}$$

Similarly, we obtain the relation

$$\begin{aligned} &M_{11}^\dagger M_{12} + M_{21}^\dagger M_{22} - (M_{11}^\dagger M_{11} + M_{21}^\dagger M_{21})\rho_R \\ &+\rho_R(M_{12}^\dagger M_{12} + M_{22}^\dagger M_{22}) - \rho_R(M_{12}^\dagger M_{11} + M_{22}^\dagger M_{21})\rho_R = 0 \ . \end{aligned} \tag{A.11}$$

Eliminating M_{22} from (A.5) and (A.6), we obtain

$$\begin{aligned} \rho_L M_{22} \rho_R^\dagger &= (M_{11}\rho_R + \rho_L M_{21}\rho_R - M_{12})\rho_R^\dagger \\ &= \rho_L(\rho_L^\dagger M_{11} + \rho_L^\dagger M_{12}\rho_R^\dagger - M_{21}) \ , \end{aligned} \tag{A.12}$$

so that

$$(1 + \rho_L\rho_L^\dagger)(M_{11} + M_{12}\rho_R^\dagger) = (M_{11} + \rho_L M_{21})(1 + \rho_R\rho_R^\dagger) \ . \tag{A.13}$$

Similarly, eliminating M_{11} from (A.5) and (A.6), we obtain

$$\begin{aligned} \rho_L^\dagger M_{11}\rho_R &= \rho_L^\dagger(M_{12} + \rho_L M_{22} - \rho_L M_{21}\rho_R) \\ &= (M_{21} + M_{22}\rho_R^\dagger - \rho_L^\dagger M_{12}\rho_R^\dagger)\rho_R \ , \end{aligned} \tag{A.14}$$

so that

$$(1 + \rho_L^\dagger\rho_L)(M_{22} - M_{21}\rho_R) = (M_{22} - \rho_L^\dagger M_{12})(1 + \rho_R^\dagger\rho_R) \ . \tag{A.15}$$

By using the relations (A.13) and (A.15), we obtain

$$\begin{aligned} M_{11}' &= (1 + \rho_L\rho_L^\dagger)^{-1/2}(M_{11} + M_{12}\rho_R^\dagger + \rho_L M_{21} + \rho_L M_{22}\rho_R^\dagger)(1 + \rho_R\rho_R^\dagger)^{-1/2} \\ &= (1 + \rho_L\rho_L^\dagger)^{-1/2}(M_{11} + \rho_L M_{21})(1 + \rho_R\rho_R^\dagger)^{+1/2} && (A.16) \\ &= (1 + \rho_L\rho_L^\dagger)^{+1/2}(M_{11} + M_{12}\rho_R^\dagger)(1 + \rho_R\rho_R^\dagger)^{-1/2} \ , && (A.17) \\ M_{22}' &= (1 + \rho_L^\dagger\rho_L)^{-1/2}(\rho_L^\dagger M_{11}\rho_R - \rho_L^\dagger M_{12} - M_{21}\rho_R + M_{22})(1 + \rho_R^\dagger\rho_R)^{-1/2} \\ &= (1 + \rho_L^\dagger\rho_L)^{+1/2}(M_{22} - M_{21}\rho_R)(1 + \rho_R^\dagger\rho_R)^{-1/2} && (A.18) \end{aligned}$$

$$= (1+\rho_L^\dagger\rho_L)^{-1/2}(M_{22}-\rho_L^\dagger M_{12})(1+\rho_R^\dagger\rho_R)^{+1/2} \ . \tag{A.19}$$

The matrices ρ_L and ρ_R are obtained as solutions of the equations (A.10) and (A.11), respectively. When the $2n\times 2n$ mass matrix M (A.1) is Hermitian, we can set $\rho_L=\rho_R\equiv\rho$, so that the calculation becomes easier.

When M is not Hermitian, instead of the $n\times n$ mass matrices M'_{11} and M'_{22}, the diagonalization is done for the following Hermitian matrices

$$H_1 \equiv M'_{11}M'^\dagger_{11} = (1+\rho_L\rho_L^\dagger)^{-1/2}\widetilde{H}_1(1+\rho_L\rho_L^\dagger)^{+1/2} \ , \tag{A.20}$$

$$H_2 \equiv M'_{22}M'^\dagger_{22} = (1+\rho_L^\dagger\rho_L)^{+1/2}\widetilde{H}_2(1+\rho_L^\dagger\rho_L)^{-1/2} \ , \tag{A.21}$$

where

$$\widetilde{H}_1 = (M_{11}+\rho_L M_{21})(M_{11}^\dagger+\rho_R M_{12}^\dagger) \ , \tag{A.22}$$

$$\widetilde{H}_2 = (M_{22}-M_{21}\rho_R)(M_{22}^\dagger-M_{12}^\dagger\rho_L) \ . \tag{A.23}$$

We are interested in the diagonalization of (A.22). By using (A.5), we can rewrite (A.22) into

$$\widetilde{H}_1 = A+\rho_L B \ , \tag{A.24}$$

where

$$A = M_{11}M_{11}^\dagger + M_{12}M_{12}^\dagger \ , \tag{A.25}$$

$$B = M_{21}M_{11}^\dagger + M_{22}M_{12}^\dagger \ . \tag{A.26}$$

By eliminating ρ_L from (A.10) and (A.24), we find that the matrix $\widetilde{H}_1$ satisfies the following equations

$$\widetilde{H}_1^2 - \widetilde{H}_1(A+B^{-1}DB) + AB^{-1}DB - CB = 0 \ , \tag{A.27}$$

where

$$C = M_{11}M_{21}^\dagger + M_{12}M_{22}^\dagger \ , \tag{A.28}$$

$$D = M_{21}M_{21}^\dagger + M_{22}M_{22}^\dagger \ . \tag{A.29}$$

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Figure Captions

Fig. 1. Masses m_i^f $(i = 1, 2, 3)$ versus b_f for the case of $k = 10$ and $K/k = 50$. The solid and broken lines denote for the cases of $\beta_f = 0$ and $\beta_f = -20°$, respectively. The parameters k and K are defined by (2.1). The figure should be taken as that for the quark mass ratios. For the absolute value of quark masses, see a comment on (4.5) in the text.

Fig. 2. Mass ratio m_t/m_c versus K/k for $k = 10$. The curves (a) – (d) denote the cases (a) $\Delta b_u = 0$, (b) $\Delta b_u = +1.00 \times 10^{-3}$ and $\Delta b_u = -0.980 \times 10^{-3}$, (c) $\Delta b_u = +3.88 \times 10^{-3}$ and $\Delta b_u = -3.62 \times 10^{-3}$, (d) $\Delta b_u = +10.0 \times 10^{-3}$ and $\Delta b_u = -8.53 \times 10^{-3}$. The horizontal lines denote the experimental values $(m_t/m_c)_{exp} = 271 \pm 46$.

Fig. 3. Top quark mass m_t in unit of m_0 versus K/k for $k = 10$. The curves (a) – (d) denote the cases (a) $\Delta b_u = 0$, (b) $\Delta b_u = +1.00 \times 10^{-3}$ and $\Delta b_u = -0.980 \times 10^{-3}$, ($c_+$) $\Delta b_u = +3.88 \times 10^{-3}$, ($c_-$) $\Delta b_u = -3.62 \times 10^{-3}$, ($d_+$) $\Delta b_u = +10.0 \times 10^{-3}$, and ($d_-$) $\Delta b_u = -8.53 \times 10^{-3}$.

Fig. 4. Mass ratios m_s/m_d and m_b/m_s versus β_d for $b_d = -0.90$ (a dotted line), $b_d = -1.0$ (a solid line) and $b_d = -1.1$ (a broken line) in the case of $k = 10$ and $K/k = 50$.

Fig. 5. Kobayashi-Maskawa matrix elements $|V_{us}|$, $|V_{cb}|$ and $|V_{ub}|$ versus β_d in the case of $k = 10$, $K/k = 50$, $b_u = -1/3$ and $\beta_u = 0$.